

## Workshop 3: Extra logic questions

1. Use a truth table to show that the proposition  $p \vee (q \vee \neg p)$  is a tautology (always true).

2. Simplify using the laws of logic:

(a)  $\neg(\neg a \wedge \neg b)$

(b)  $(a \wedge b) \vee (a \wedge \neg b) \vee (\neg a \wedge b)$

(c)  $(q \wedge \neg p) \vee p$

3. Use laws of logic to show that  $\neg((p \vee \neg q) \vee (r \wedge (p \vee \neg q)))$  is equivalent to  $\neg p \wedge q$ .

4. Use laws of logic to prove that  $(z \wedge w) \vee (\neg z \wedge w) \vee (z \wedge \neg w)$  is equivalent to  $z \vee w$ .

5. Which of the following are tautologies? Show using laws of logic.

(a)  $p \Rightarrow (p \vee q)$

(b)  $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$

(c)  $(p \wedge (p \Rightarrow q)) \Rightarrow q$

(d)  $(p \wedge q) \Rightarrow p$

(e)  $q \Leftrightarrow (\neg p \vee \neg q)$

6. Consider the following predicates:

$$P(x,y) : x > y$$

$$Q(x,y) : x \leq y$$

$$R(x) : x - 7 = 2$$

$$S(x) : x > 9$$

What is the truth value of the following:

(a)  $\exists x \in \mathbb{N} : R(x)$

(b)  $\forall y \in \mathbb{N} : S(y)$

(c)  $\forall x \in \mathbb{N} : \exists y \in \mathbb{N} : P(x,y)$

(d)  $\exists y \in \mathbb{N} : \forall x \in \mathbb{N} : Q(x,y)$

(e)  $\forall x \in \mathbb{N} : \forall y \in \mathbb{N} : P(x,y) \vee Q(x,y)$

(f)  $(\exists x \in \mathbb{N} : S(x)) \wedge \neg(\forall x \in \mathbb{N} : R(x))$

7. Write the following in predicate logic:

(a) Some people drink coffee.

(b) All programmers drink coffee.

(c) There exists some programmers who write good code but do not drink coffee.

(d) For all people, if they don't drink coffee then they write bad code.

Use the predicates:  $P(x)$  :  $x$  is a programmer    $C(x)$  :  $x$  drinks coffee    $G(x)$  :  $x$  writes good code