Workshop 5: Relations

1. Which of the following sets satisfy the definition of a relation? Explain why.
    a. {{x,y},{y,z},{z,v}}
    b. {(2,3)}
    c. {(1,3,2),(3,2,4)}

2. For the following relations, say whether they are reflexive, symmetric, antisymmetric, or transitive?
    a. $R_>$
    b. ∅
    c. R_p : People ↔ People where a p b means a and b have the same birthday
    d. $R_|$: $\mathbf{N} \leftrightarrow \mathbf{N}$ where m | n means m divides n (or $\exists k \in \mathbf{N} : km = n$)

3. Let R be the relation {(1,2), (1,3), (2,3), (2,4), (3,1)} and let S be the relation {(2,1), (3,1), (3,2), (4,2)}. Find R ○ S.

4. Consider the relation $R_{letters}$: Words ↔ Words
    where a letters b ⇔ a has the same number of letters as b.
    a. Show that $R_{letters}$ is an equivalence relation.
    b. How many equivalences classes does $R_{letters}$ have?

5. Consider two relations $R_p : A \leftrightarrow A$ and $R_q : A \leftrightarrow A$. Which of the following statements are true and why?
    a. If R_p is symmetric then $R_{p^{-1}}$ is also symmetric.
    b. If R_p and R_q are symmetric then $R_p \cap R_q$ is symmetric.
    c. If R_p and R_q are transitive then $R_q \cup R_p$ is transitive.
    d. If R_p and R_q are reflexive then $R_p \cup R_q$ is reflexive.