Workshop 6: Functions (and more relations)

1. Which of the following relations are reflexive, symmetric, antisymmetric or transitive? Explain why.

    a. $R_a : \{1,2,3,4\} \leftrightarrow \{1,2,3,4\}$
       $R_a = \{ (1,1), (2,2), (3,3) \}$

    b. $R_b : \{1,2,3,4\} \leftrightarrow \{1,2,3,4\}$
       $R_b = \{ (1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3) \}$

    c. $R_c : \mathbf{N} \leftrightarrow \mathbf{N}$
       $R_c = \{ (x,y) \in \mathbf{N} \times \mathbf{N} \mid y = x + 1 \}$

2. Consider the relation $R_{letters} : \text{Words} \leftrightarrow \text{Words}$ (again!) where two words are related if they have the same number of letters.

    This week, let Words = { Hello, Tata, chickens, love, discrete, maths, goodbye }.

    a. Draw a diagram of this relation.
    b. How many equivalence classes are there in this relation?

3. In the set Words, change "Tata" for your own name.

    a. Draw a new diagram for this relation.
    b. Write down every equivalence class.

4. Which of the relations in question 1 are functions?

5. Which of the following functions are surjective, injective, or bijective?

    a. $f : \mathbf{N} \to \mathbf{N}$
       $f(x) = x^2$

    b. $f : \{1,2,3\} \to \{1,2,3\}$
       $f = \{ (1,2), (2,3), (3,1) \}$

    c. $f : \{a,b,c,d\} \to \{milk, toast, cornflakes\}$
       $f = \{ (a,milk), (b,milk), (c,toast), (d,toast) \}$

    d. $f : \mathbf{N} \to \mathbf{N}$
       $f(x) = floor(x/2)$ (the floor function takes any real number and returns the integer part)

    e. $f : \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$
       $f(x,y) = x + y$