Workshop 7: Induction

1. Let P(n) be the statement $1^2 + 2^2 + 3^2 + ... + n^2 = n(n+1)(2n + 1)/6$ where n is a natural number.

   a. What is the value of P(4)?

   b. Show that the base case P(0) is true.

   c. Write the inductive step.

   d. Show that the inductive step is true.

2. Using proof by induction, show that $1 + 3 + 5 + ... + (2n - 1) = n^2$ is true for all positive integers.

3. Prove that $1*2 + 2*3 + 3*4 + ... + n(n+1) = n(n+1)(n+2)/3$ is true for all positive integers.

4. Using mathematical induction, prove that $2^n < n!$ (! = factorial) for all positive integers greater than 3 ($n > 3$).

5. Write a recursive function for SumSequence(n) = 1 + 2 + 3 + ... + n. For example:
   SumSequence(1) = 1 SumSequence(2) = 3 SumSequence(3) = 6

6. Write a recursive function that returns the *n*th Fibonacci number. For example:
   Fibonacci(1) = 1 Fibonacci(2) = 1 Fibonacci(3) = 2 Fibonacci(4) = 3

7. Write an *iterative* function that returns the *n*th Fibonacci number.

Bonus question (for real computer scientists!)

8. The following function, called Search, is similar to the recursive function Contains (from the lecture).

```
int Search(int[] array, int x, int startposition) {

    ???

}
```

   Implement the function Search such that it returns the position of the number x in the array (or -1 if not found). This is called a linear search. For example:
   Search({2,5,6,3,4},5,1) = 2 Search({2,5,6,3,4},4,1) = 5 Search({2,5,6,3,4},8,1) = -1